STUDENT MISCONCEPTIONS IN SIGNALS AND SYSTEMS AND THEIR ORIGINS

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Abstract *— We report on our ongoing investigation on student misconceptions and their origins within the Signals and Systems module taught in the Department of Aeronautics and Astronautics at the Massachusetts Institute of Technology. Signals and Systems, as taught in Aeronautics and Astronautics at MIT, consists of two parts. The first part, offered in the Fall semester, covers introductory linear circuits; the second part, offered in the Spring semester, covers the analysis of generic continuous-time linear time-invariant systems. During Fall 2002, we conducted clinical interviews to assess student understanding of introductory linear circuits. Fifty-four sophomore students enrolled in Signals and Systems volunteered to take part in this study. The interview transcripts were analyzed, physical and mathematical misconceptions were identified, and their sources were examined based on diSessa's theory of intuitive knowledge, and Chi and Slotta's ontological categorization. In this paper, we report on our results and suggest how this understanding can be used to develop more effective pedagogical instruments designed to enhance student learning.*

Index Terms — Active learning, signals and systems, misconceptions, phenomenological primitives, ontological categorization.

INTRODUCTION

Since 1999, professors in the Department of Aeronautics and Astronautics at the Massachusetts Institute of Technology (MIT) have been implementing active learning techniques (*e.g.*, concept tests, muddiest-point-in-the-lecture) to support student learning. (See, for example, [14].) One of us (Hall) has been using active techniques in the Signals and Systems module taught in the department. However, one of the difficulties we have encountered is that there is little scholarly literature on misconceptions in the signals and systems discipline. This had made it difficult to develop effective active learning materials, such as concept tests, that depend on understanding typical student misconceptions. Some misconceptions can be uncovered in the field, in the course of normal teaching activities, or by student responses on mud cards. However, it is unclear whether these techniques are powerful enough to uncover important misconceptions that inhibit student learning. Therefore, we undertook a more rigorous study whose purpose is to determine student misconceptions in signals and systems, and whose meta-purpose is to determine whether a rigorous study is in fact required to determine student misconceptions, or whether more informal means (such as mud cards) can yield the same information.

The Signals and Systems module, as taught in the Department of Aeronautics and Astronautics at MIT, involves learning concepts and algorithms for the analysis of linear electrical circuits and generic continuous-time, linear, time-invariant systems. As a discipline, signals and systems is in a large part detached from daily experience and significantly embedded in abstract mathematical modeling. Despite the ubiquity of electricity in everyday life, electrical circuitry remains, even in its simplest structures, significantly abstract for students to comprehend. Even after repeated instruction, basic electricity concepts such as potential, potential difference, and capacitance continue to be stumbling blocks for students. Furthermore, signals and systems relies heavily on higher-level mathematics, especially calculus and differential equations. Students generally find difficulties and hold misconceptions in these mathematical domains [1, 2]. These could hinder the understanding of signals and systems by feeding into physical misconceptions and by constraining a valid transfer between the physical model and its mathematical representation. It is thus of interest to undertake a structured and in-depth investigation of students' misconceptions in signals and systems and their physical and mathematical cognitive resources that generate these misconceptions.

The physics education literature contains a wealth of research on students' conceptual understanding in varied domains, such as mechanics, thermodynamics, optics, and electricity [3]. For instance, considering the physics domain pertinent to this study (electricity), numerous studies have been conducted to elucidate student understanding of simple electric circuits. Some of the misconceptions and difficulties that have been documented include: Failure to differentiate between concepts of current, energy, and power, and potential and potential difference [7]; Belief that current flow is a sequential process that has a beginning and an end [4]; Belief that current gets used up as it flows through the elements in a circuit [5], [7]; Belief that the current through a given circuit element is not affected by the circuit modification introduced after that element [4]; Belief that a battery is a constant current source [5, 6]; Misinterpretation of Ohm's law [5, 6]; Failure to recognize that an ideal voltage source maintains a constant potential difference between its terminals [7]; and Difficulty

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identifying series and parallel connections [7].

In contrast, little research has been done on student misconceptions in engineering disciplines. Particularly for signals and systems, only Wage et al.'s [8] work-in-progress has been documented in the engineering education literature.

Moreover, a large portion of the studies on misconceptions has sought merely to identify student misconceptions. Smith *et al*. [9] argues that research that simply documents misconceptions in another domain will not advance our understanding, and that it is imperative to redirect the emphasis in research from simply documenting misconceptions to investigating their genesis.

In this paper, we discuss the theoretical frameworks grounding this investigation on student misconceptions and their origins. We describe our methodology and present our analysis of the data collected in Fall 2002 on student understandings of linear electric circuits. Finally, we conclude by drawing implications for pedagogy.

THEORETICAL FRAMEWORK

The origin of student misconceptions lies in the context of other coexisting ideas or what Strike and Posner call a "conceptual ecology" [10]. They define a conceptual ecology as consisting of such cognitive artifacts as analogies, metaphors, epistemological and metaphysical beliefs, scientific conceptions and misconceptions, and knowledge from other areas of inquiry. Smith *et al.* [9] and Hammer [11] referred to these components as *resources* meaning "any feature of the learner's present cognitive state that can serve as significant input to the process of conceptual growth" [9]. According to Strike and Posner, all elements of a conceptual ecology are in constant interaction either hindering or supporting an individual's learning, depending on their character. Based on Strike and Posner's theory, this study aims at characterizing the components of students' conceptual ecologies that hinder their learning of signals and systems and generate misconceptions.

One of the prominent frameworks on the origins of misconceptions that have been proposed in the science education literature is diSessa's [12] model of intuitive knowledge. According to diSessa, naive conceptions are the product of a fragmented set of primitive mental constructs that he calls *phenomenological primitives*, or p-prims. These are fundamental pieces of intuitive knowledge developed as a result of one's experience with the world. They are context-free constructs that are abstracted from prior experience and employed to rationalize other phenomena. According to diSessa, a misconception is generated by faultily activating a p-prim, or a set of p-prims, in the inappropriate context.

For instance, when asked to explain why it is hotter in the summer than it is in the winter, many students reason, based on their knowledge that the earth orbits elliptically around the sun, that this is the result of the earth being closer to the sun during the summer [11]. It is possible that this misconception is not existent in the students' minds prior to asking the question, but that the students intuitively construct it at the instant the question is asked by misapplying the p-prim relating intensity with proximity: closer means stronger. Students abstract this conceptual resource from a number of daily phenomena, such as the closer you are to the speaker the louder the music, or the closer you are to a bulb the more intense is the light [11].

Another theoretical framework that has been proposed to account for the origin of student misconceptions is Chi and Slotta's [13] theory of ontological categorization. Chi and Slotta concur with many of the major assertions of diSessa's theory; however, they challenge diSessa's premise that intuitive knowledge (phenomenological primitives) is fragmented. They attempt to extend his theory by adding another cognitive dimension of ontological categories that would provide coherence and structure to intuitive knowledge.

Chi and Slotta's theory is based on an epistemological premise that all entities in the world can be classified into ontological categories according to their ontological attributes. Their theory accommodates phenomenological primitives as instances that reflect ontological attributes. Chi and Slotta propose three primary categories: matter, processes, and mental states, and within each of these categories, they structure a hierarchy of sub-categories [13].

Chi and Slotta argue that a misconception is the product of relegating a concept to an inappropriate ontological category. Once a concept is misclassified under a category, it becomes associated with that category's pool of ontological attributes [13]. For example, student misconceptions regarding the concept of heat are the result of students classifying heat as a material flowing substance that can be "blocked" or "contained" rather than as a process of molecular excitation [13]. "Blocking" would be the p-prim that corresponds to the ontological attribute or verbal predicate "blocks," which in turn is associated with the substance ontology.

METHODOLOGY

For this study, clinical interviews were the primary mode of inquiry we used to probe students' understanding of Signals and Systems. Interviews are a powerful method for capturing the crucial characteristics of a person's knowledge and the fluidity of his or her thinking [16]. They are generally recognized as the most effective means for understanding a subject's state of knowledge.

Signals and Systems is a part of a larger fundamental engineering course, Unified Engineering, that is offered as a requirement for sophomore students in Aeronautics and Astronautics Department at MIT. Signals and Systems, as taught in Unified, consists of two parts: The first part, covered during the first five weeks of the Fall semester, involves the analysis of linear electrical circuits. The second part, offered during the last eight weeks of the Spring semester, involves the analysis of generic continuous-time linear systems. There are a total of approximately 40 one-hour lectures in Signals and Systems. Students enrolled in Unified are required to take a course in differential equations prior to or during the Fall semester of their enrollment in Unified. Also, the course Physics II is a prerequisite to Unified. Physics II is an

introduction to electromagnetism and electrostatics.

In Fall 2002, oral problems were introduced as part of the requirements in Signals and Systems. The students were divided into four cohorts, and were interviewed individually over the course of four weeks. The students in each cohort worked the same problem. Each student was scheduled for a one-hour oral problem session. During the first half hour, the student was given the problem statement, and allowed to prepare a preliminary answer in private. During the second half hour, the student sat with the course instructor, who probed his or her understanding of the problem. All 70 students enrolled in Unified Engineering were required to do one oral problem. The 54 students who volunteered to participate in this study had their oral sessions audio-taped and/or video-taped.

In this paper, we will report only on our analysis of student responses to the first and third problems. These two problems were selected because they elicited the most serious mathematical and physical misconceptions. The interview transcripts were analyzed by coding students' misconceptions and difficulties in physics and mathematics and identifying their sources. The sources that were identified generally involve students' ontological perceptions of the various physical concepts, and other cognitive resources such as phenomenological primitives or ontological attributes that elucidate students' ontological perspectives.

RESULTS

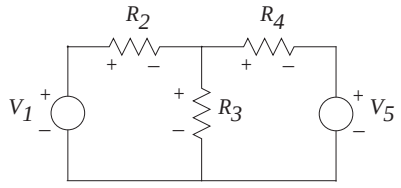
Oral Problem 1

Oral Problem 1 is shown in Figure 1. The problem tests students understanding of simple linear resistive networks. In the interviews, students were asked to explain their general approach to the problem and to explain the meaning of the equations that they used. Students exhibited difficulty with the meaning of standard sign conventions, proper application of the node method to solve the network, and the meaning of potential.

By convention, the plus and minus signs on the resistors in the circuit of Oral Problem 1 do *not* indicate which terminal of the resistor is at the higher potential. Rather, the signs indicate how the potential is to be measured across the resistor. For example, if the potential of the positive terminal of R_2 is lower than the potential of the negative terminal, then v_2 will be negative. However, some students believe that regardless of the reference polarity assigned to a circuit element, the voltage across the element must be expressed as a positive number, as seen in the following exchange:

> Student: Well, it's [the voltage across R_4] 4 volts because you can't have a negative voltage. It was negative because I was doing it with relation to the node.
>
> Professor: But for example, if this were not just a homework problem but an exam, and I said write in the space below "$v_4 = \ldots$" what would you put there?
>
> S: Then I would work it out on some scratch paper and then I would put 4V there.

Consider the circuit below:



where $V_1 = 3$ V, $R_2 = 3\ \Omega$, $R_3 = 6\ \Omega$, $R_4 = 2\ \Omega$, and $V_5 = 10$ V.

1. Find the voltage across each element, and the current through each element.
2. Which elements dissipate or absorb power? Which elements supply power? Explain.

FIGURE 1. ORAL PROBLEM 1.

> P: Even with the plus and minus sign here.
>
> S: Yes.

This particular misconception is one of the few that we had correctly predicted would appear in student interviews. Indeed, using the muddiest-point-in-the-lecture method of Mosteller [15], we had previously seen this misconception expressed in student "mud cards." As a result, the lectures in Unified emphasized the correct meaning of the signs. Despite this treatment, the misconception persisted, indicating the resilience of this particular misconception.

Students were taught two (equivalent) procedures for solving circuits using the node method: In the first method, a student expresses Kirchhoff's current law at each node with unknown voltage in terms of the current variables according to the assigned sign convention. For Oral Problem 1, the result is

$$-i_2 + i_3 + i_4 = 0 \tag{1}$$

The student then expresses each current using the appropriate constitutive law (Ohm's law for resistors), in terms of known and unknown node potentials. For this problem, most students labeled the bottom node connecting V_1, R_3, and V_5 as ground, and the node connecting the three resistors as e_1. In terms of the potentials, the node equation becomes

$$-\frac{v_1 - e_1}{R_2} + \frac{e_1 - 0}{R_3} + \frac{e_1 - v_5}{R_4} = 0 \tag{2}$$

In the second approach, students apply the "near-minus-far" rule. That is, they immediately write the current flowing out of a node through a resistor as the potential of the near terminal of the resistor minus the potential of the far terminal, all divided by the resistance. In this case, the node equation is then

$$\frac{e_1 - v_1}{R_2} + \frac{e_1 - 0}{R_3} + \frac{e_1 - v_5}{R_4} = 0 \tag{3}$$

which is the same as Equation (2).

A commonly committed error was the application of the near-minus-far rule with an incorrect interpretation of the sign

convention, typically resulting in the (incorrect) node equation

$$-\frac{e_1 - v_1}{R_2} + \frac{e_1 - 0}{R_3} + \frac{e_1 - v_5}{R_4} = 0. \quad (4)$$

Some students had difficulty recognizing that the resulting node equation, using either approach, is independent of the labeling of the $+/-$ signs in the circuit. Instead, students often conflated the two approaches, leading to sign errors as in Equation (4) above. When students were asked to explain why they had a minus sign in front of the first term in their (incorrect) node equation, this confusion was evident:

> S: i_2 was going into the node, so I was summing all the currents out of the node, so i_2 would be negative.
>
> P: So why did you write inside the parentheses $e_1 - v_1$?
>
> S: I guess because I was thinking near minus far.

and

> S: The minus, it's going in the opposite direction as the i_2 here, so that's where I get the minus for that. And here too, it's going from plus to minus out of the node. So that's where I get the plusses from.

The near-minus-far rule was introduced to simplify the task of producing node equations. Indeed, if applied correctly, the method does significantly reduce the effort required to generate node equations. However, students had difficulty applying the rule correctly, especially when $+/-$ signs were shown on the circuit. Anecdotally, we can report that students had less difficulty when the $+/-$ signs were left off the circuit diagram. Apparently, the signs provide a strong (and incorrect) cue to some students, even when they can correctly apply the procedure in the absence of the signs.

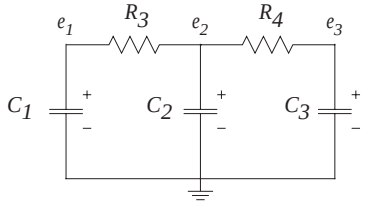
Students who incorrectly applied the near-minus-far rule found the node equation to be as shown in Equation (4). These students found e_1 to be 12V. This result cannot be correct: Given that $V_1 = 3$V, $V_2 = 10$V, and that both sources have their negative terminal connected to ground, it is not possible (in a resistive network) for the potential anywhere in the circuit to be greater than 10V. When pressed on this point, students constructed the erroneous idea that "potential accumulates" to physically explain their solution. When asked whether it is plausible to get 12 volts somewhere in the circuit, one student responded

> S: It seems like you could ... It seems like the voltages could sum ... I guess the maximum would be 13 volts.

Another student appealed to a water-flow analogy to explain his answer. However, he misapplied the analogy due to his incorrect understanding of the concept of pressure, which hindered a correct transfer to the voltage concept:

> S: You got stuff converging this way and pushing pressure on both ends toward the node there ... This [V_1] is pushing, you know, adding potential this way and this [V_5] is adding potential this way ... so I was thinking you might end up with a higher [potential] ... if you just add them you just end up with 13 ...

Consider the circuit below:



where $C_1 = 1$ F, $C_2 = 2$ F, $C_3 = 1$ F, $R_4 = 1\ \Omega$, and $R_5 = 1\ \Omega$. Please be prepared to answer the following questions:

1. Find a set of differential equations that describes the node voltages as a function of time.
2. Find the characteristic values of the system.
3. Find the characteristic vectors of the system.
4. If the initial conditions are $v_1(0) = 4$ V, $v_2(0) = 0$ V, and $v_3(0) = 0$ V, what is $e_1(t)$?

FIGURE 2. ORAL PROBLEM 3.

Evidently, these students ascribed an incorrect ontology to the concepts of potential and pressure. They dealt with these concepts not as scalar fields but as "stuff," or substance-like entities that could be pushed and accumulated. This is reflected in the way they utilized two ontological attributes: potential/pressure can be "pushed," and potentials/pressures "sum up," which in turn correspond to the p-prims "pushing" and "accumulation," respectively.

Oral Problem 3

Oral Problem 3 is shown in Figure 2. The question tests students understanding of capacitive networks, and their ability to determine the differential equations that describe the evolution of the circuit over time. During the interviews, students were asked to explain their mathematical approach to solving this problem and to qualitatively describe the physical behavior of the circuit as it evolves from its transient state to its steady state.

Despite the complexity of the problem, many students had the correct answer to the problem as formulated when they arrived at the interview. Nevertheless, some of these students were unable to give a valid qualitative description of the transient behavior of the circuit. First, some students were unable to relate the characteristic values and vectors they obtained to the physical behavior of the circuit, and hence they were not used as a resource in constructing their physical analysis of the system. As one student commented, "It's one thing to do the math and another thing to actually understand it." Second, some students had an inadequate understanding of the concept of potential. As discussed earlier, they did not construe the potential as a field, but rather as a substance-like entity. This is evident from the ontological attributes they ascribe to "voltage" that are manifested in students' explanations as p-prims.

A typical misconception that was elicited in students' reasoning is that voltage is conserved. For instance, consider the excerpt below:

> S: Capacitor 1 is the only one that is charged and e_2 and e_3 are not charged. Then the charge from capacitor one will disperse throughout the circuit onto C_2 and C_3. And at steady state, they all should be equal since they're all in parallel ... So that means you're going to take 4V from C_1 and disperse it across three different capacitors equally. So at steady state, they should be at four-thirds for each one.

This misconception is the product of the activation of the "conservation" p-prim. Students seem to have overgeneralized an otherwise useful concept when applied to other physical quantities such as mass, energy, and charge. Their attribution of "conservation" to the concept of voltage is due to their ontological misclassification of voltage as a substance.

Students who presented this argument did not appeal to the law of conservation of charge. They were encouraged to reconsider their reasoning by bringing to their attention the fact that capacitor C_2 has a larger capacitance than C_1 or C_3. This elicited another misconception that voltage is directly proportional to capacitance:

> S: That means the [second] capacitor would have twice the voltage as [capacitors] 1 and 3.

Students directly appealed to simple linear reasoning without referring to the concept of charge or to the constitutive law $q = Cv$. The p-prim "more capacity means more is stored" was misapplied in this situation. Instead of invoking the concept of charge, the students incorrectly employed the concept of voltage. Again, this is due to a misunderstanding of the concept of voltage resulting from an incorrect ontological conception of voltage. Rather than construing voltage as a field, the students seem to understand voltage as a measure of a quantity of charge.

Further, these students' reasoning indicates that students intuitively focus on two variables in their analysis. They do not simultaneously invoke all the concepts of current, charge, voltage, and capacitance and their interdependence to analyze the system. For example, they did not realize that if e_2 is twice e_1 and e_3 then there will be current flow and the values of e_1, e_2, and e_3 will not remain constant. When students were led to realize the flaw in their reasoning, they expressed another misconception that voltage can flow and be exchanged among circuit elements:

> S: Well it would seem that they're almost going to continue feeding each other, the capacitors. [The voltage] starts off on the one capacitor with these at zero and then it starts to flow and it increases each of the other capacitors' values ... but also it's flowing out of them as well at the same time. And they're going to continue almost exchanging the voltage.

The way students use the predicates "flow" and "exchanging" manifests their implementation of the phenomenological primitive or the substance attribute of "particulate motion." Again, students seem to conflate the field-concept of voltage with the substance-concept of charge and to construe voltage as a measure of a quantity of charge.

Other students reasoned that the voltage dissipates and that in steady-state the voltage across the capacitors is zero:

> S: I know that this [C_1] is going to dissipate the voltage because there is no voltage source and there are two resistors, but first it should initially charge up C_2 and C_3 ... And then eventually over time it should all be dissipated in the resistors.

In the absence of a source that would continuously supply voltage, the students intuitively appealed to the p-prim "dissipation" or what diSessa refers to as the "dying away" primitive. Again, students drawing on this p-prim construe voltage as a substance-like entity that gets dissipated as energy in the circuit:

> S: The charge, the voltage ... ends up leaving the circuit as heat through the resistor.

All of the above misconceptions about the concept of voltage suggest that the notion of a "field" forms an ontological obstacle for students. They apparently have difficulty understanding the nature of scalar fields, such as "potential" and "pressure." When one of the students was asked to define voltage, she could not qualitatively explain the concept. Instead she sought to construct a definition of voltage from the formula $q = Cv$. She defined voltage as that

> S: ... due to the charge on one plate of the capacitor [and] ... also related to the capacitance.

Her definition did not go beyond the mathematical dependence indicated in the equation. Students do not construe voltage (potential difference) as a difference of scalar potential between two points. This results in their drawing on their extant cognitive resources of phenomenological p-prims and ontological attributes that are essentially associated with the familiar substance ontology, and hence with the fundamental electrical substance of charge.

The constitutive law for a capacitor may be written as either

$$i = C\frac{dv}{dt} \qquad \text{or} \qquad q = Cv \tag{5}$$

where C is the capacitance, v is the voltage across the capacitor, q is the charge on the capacitor, and $i = dq/dt$ is the current through the capacitor. Generally, students were able to apply these equations correctly to determine the differential equations that describe the time evolution of the circuit. However, when using the constitutive law to explain the behavior of the circuit, many students applied the law incorrectly, often considering only two of the three variables in the law. For example, some students reasoned that in steady-state, the voltage across C_2 must be less than the voltage across C_3, since C_2 is greater than C_3. These students focused on the relationship between C and v, implicitly assuming that q is the same for all capacitors. In fact, the correct principle to

use is that, almost by definition, $dv/dt = 0$ in steady-state, so that the steady-state current through any capacitor is zero. Because of the topology of the circuit, this implies that there is no current through any resistor, and hence the voltage across each resistor is zero, and the voltage across all the capacitors must be the same. Thus, the charge on C_2 is greater than the charge on the other capacitors.

Some students had problems interpreting the differential in Equation (5). They directly invoked the "proportionality" p-prim and inferred that more capacitance means more current: More current is going to flow through the second capacitor than through the first and third capacitors. As discussed above, the net charge flowing through C_2 *is* greater than the net charge flowing through the other capacitors. In the transient, however, the current varies over time in a more complicated way. These students only considered two variables — capacitance and current — and avoided any reference to the differential term.

Other students who attempted to account for the rate of change of voltage failed to interpret the differential properly:

> S: So [the constitutive relation] just means for a given current you'll get more voltage difference between the two [capacitors].

The expression dv/dt triggered the "difference" p-prim, which was misapplied in the interpretation of the constitutive relation.

In all these cases, students were unable to translate the mathematical description of a capacitor in a mental model that allowed them to predict the transient and steady-state behavior of the circuit.

CONCLUSION

Even though we have found a wide range active learning techniques, including muddiest-point-in-the-lecture and concept tests, to be effective in revealing some student misconceptions, the results of this study indicate that these techniques have limited power to expose the breadth and depth of misconceptions. By conducting clinical interviews, we were able to uncover a more detailed set of student misconceptions and their origins. These could in turn feed into rectifying students' conceptual knowledge by adjusting and developing instruction and active learning teaching material.

In particular, instruction should be tailored to help students activate the appropriate cognitive resources for a given concept domain. It should also confront students' incorrect ontological perceptions of physical concepts that engender misconceptions. These are typically reflected in the language students use in articulating their knowledge and in the manner with which they apply analogies. Moreover, instruction should further emphasize the correspondence between the physics and the mathematical models. Even though students exhibit proficiency in performing mathematical algorithms in their analysis of physical systems, they may fail to see the mathematical-physical correspondence.

The results of this investigation could also facilitate the development of more effective multiple choice concept questions. From the identified phenomenological primitives and ontological attributes that students appeal to when talking about a particular concept, we could derive the various possible misconceptions that students tend to invoke in a specific concept domain. These could then constitute the set of distracters for a concept question in that domain.

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